Introduction to Electromagnetic Fields; Maxwell's Equations

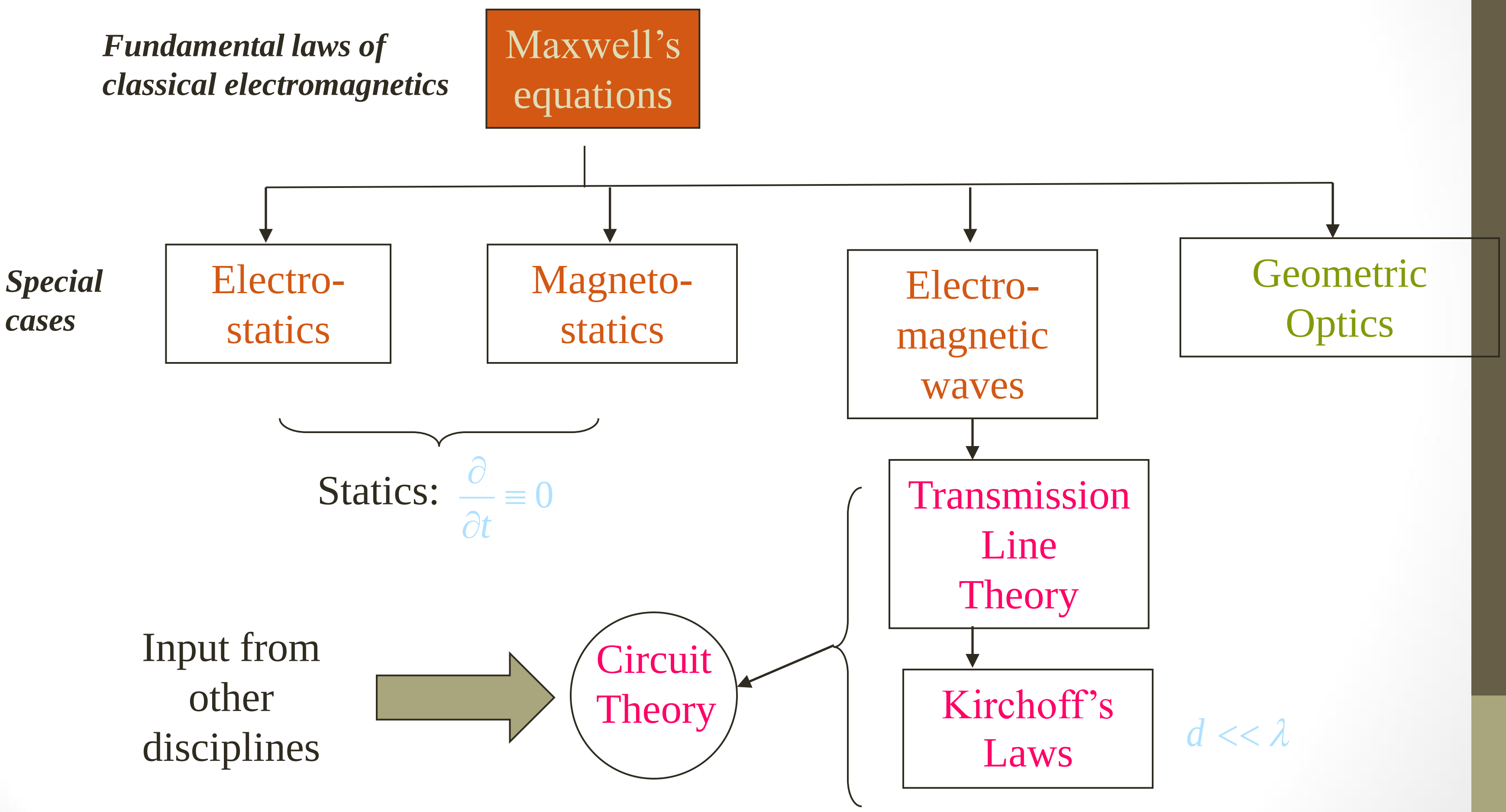
Lecture 2

- To provide an overview of classical electromagnetics, Maxwell's equations, electromagnetic fields in materials, and phasor concepts.
- To begin our study of electrostatics with Coulomb's law; definition of electric field; computation of electric field from discrete and continuous charge distributions; and scalar electric potential.

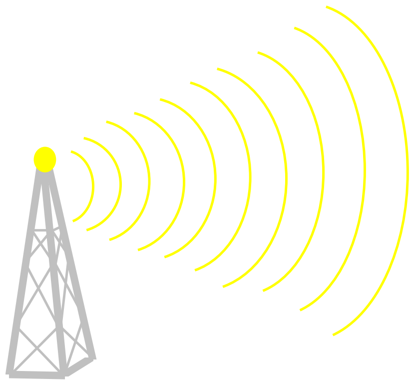
Introduction to Electromagnetic Fields

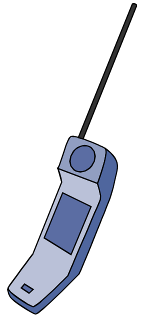
- Electromagnetics is the study of the effect of charges at rest and charges in motion.
- Some special cases of electromagnetics:
  - Electrostatics: charges at rest
  - Magnetostatics: charges in steady motion (DC)
  - Electromagnetic waves: waves excited by charges in time-varying motion

Introduction to Electromagnetic Fields



Introduction to Electromagnetic Fields

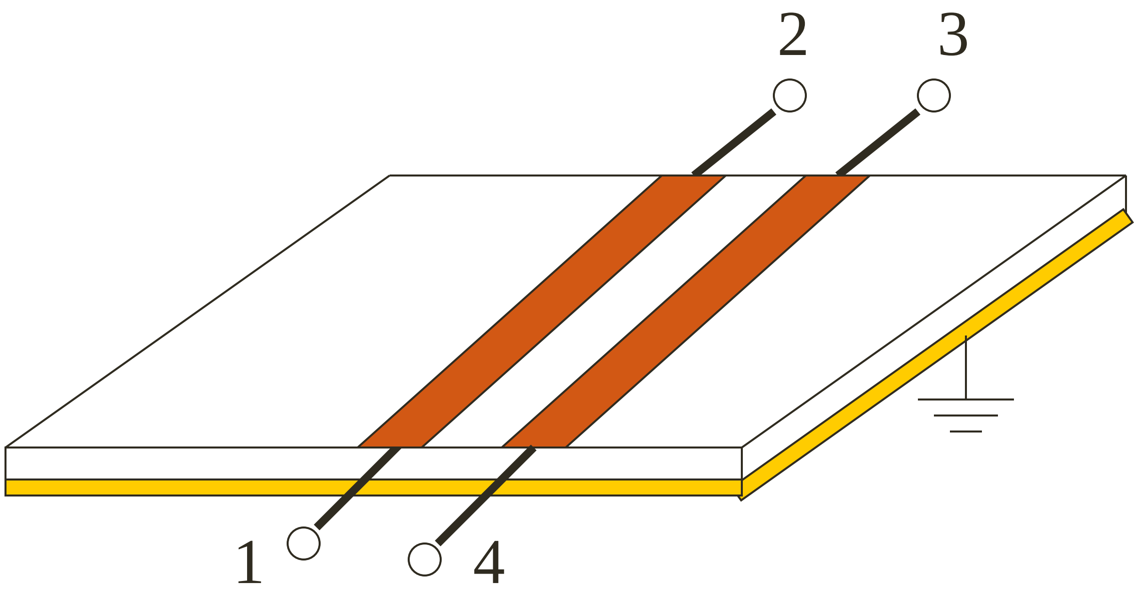




• transmitter and receiver are connected by a "field."

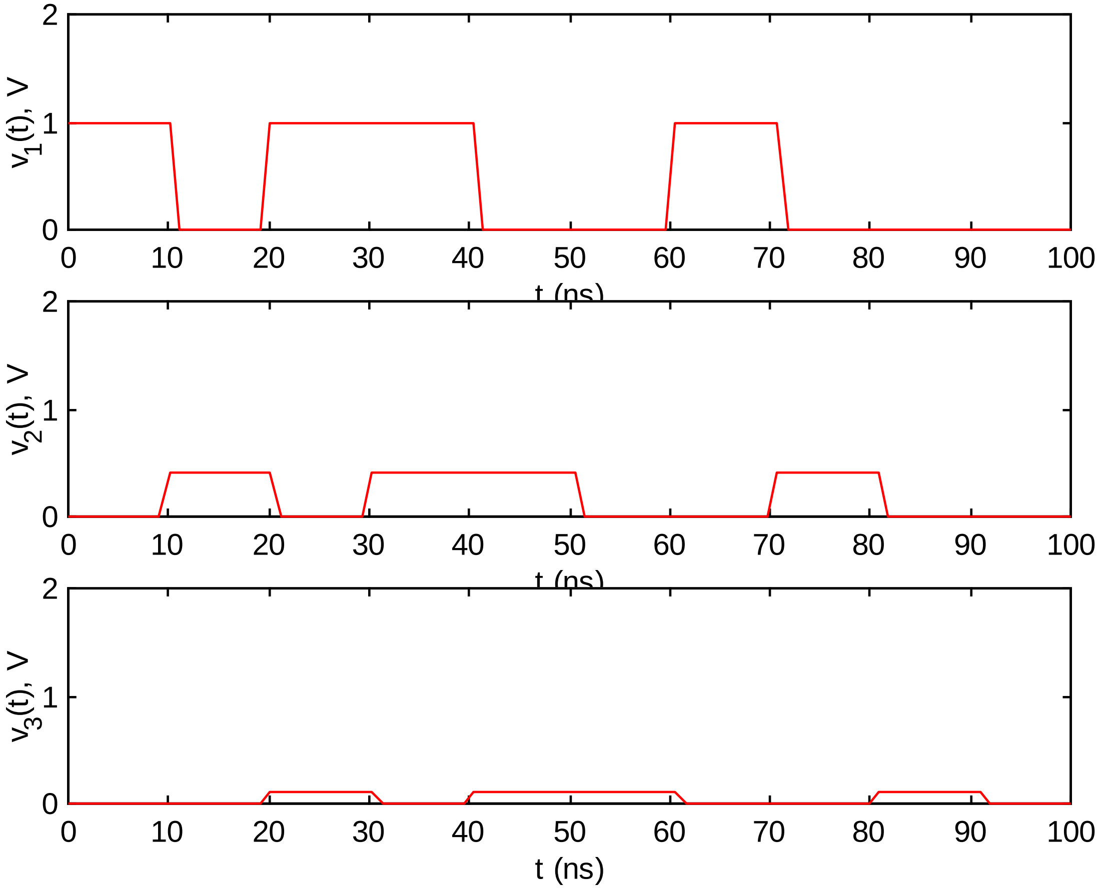
Introduction to Electromagnetic Fields

High-speed, high-density digital circuits:



- consider an interconnect between points "1" and "2"

Introduction to Electromagnetic Fields



- Propagation delay
- Electromagnetic coupling
- Substrate modes

Introduction to Electromagnetic Fields

- When an event in one place has an effect on something at a different location, we talk about the events as being connected by a “field”.
- A ***field*** is a spatial distribution of a quantity; in general, it can be either *scalar* or *vector* in nature.

Introduction to Electromagnetic Fields

- Electric and magnetic fields:
  - Are vector fields with three spatial components.
  - Vary as a function of position in 3D space as well as time.
  - Are governed by partial differential equations derived from Maxwell's equations.

Introduction to Electromagnetic Fields

- A ***scalar*** is a quantity having only an amplitude (and possibly phase).

Examples: voltage, current, charge, energy, temperature

- A ***vector*** is a quantity having direction in addition to amplitude (and possibly phase).

Examples: velocity, acceleration, force

Introduction to Electromagnetic Fields

- Fundamental vector field quantities in electromagnetics:
  - Electric field intensity $\left(\underline{E}\right)$
    units = volts per meter (V/m = kg m/A/s^3)
  - Electric flux density (electric displacement) $\left(\underline{D}\right)$
    units = coulombs per square meter (C/m^2 = A s /m^2)
  - Magnetic field intensity $\left(\underline{H}\right)$
    units = amps per meter (A/m)
  - Magnetic flux density $\left(\underline{B}\right)$
    units = teslas = webers per square meter (T = Wb/ m^2 = kg/A/s^3)

Introduction to Electromagnetic Fields

- Universal constants in electromagnetics:
  - Velocity of an electromagnetic wave (e.g., light) in free space (perfect vacuum)

$$c \approx 3\times10^{8}\ \text{m/s}$$

  - Permeability of free space

$$\mu_0 = 4\pi\times10^{-7}\ \text{H/m}$$

  - Permittivity of free space:

$$\varepsilon_0 \approx 8.854\times10^{-12}\ \text{F/m}$$

  - Intrinsic impedance of free space:

$$\eta_0 \approx 120\pi\ \Omega$$

Introduction to Electromagnetic Fields

- Relationships involving the universal constants:

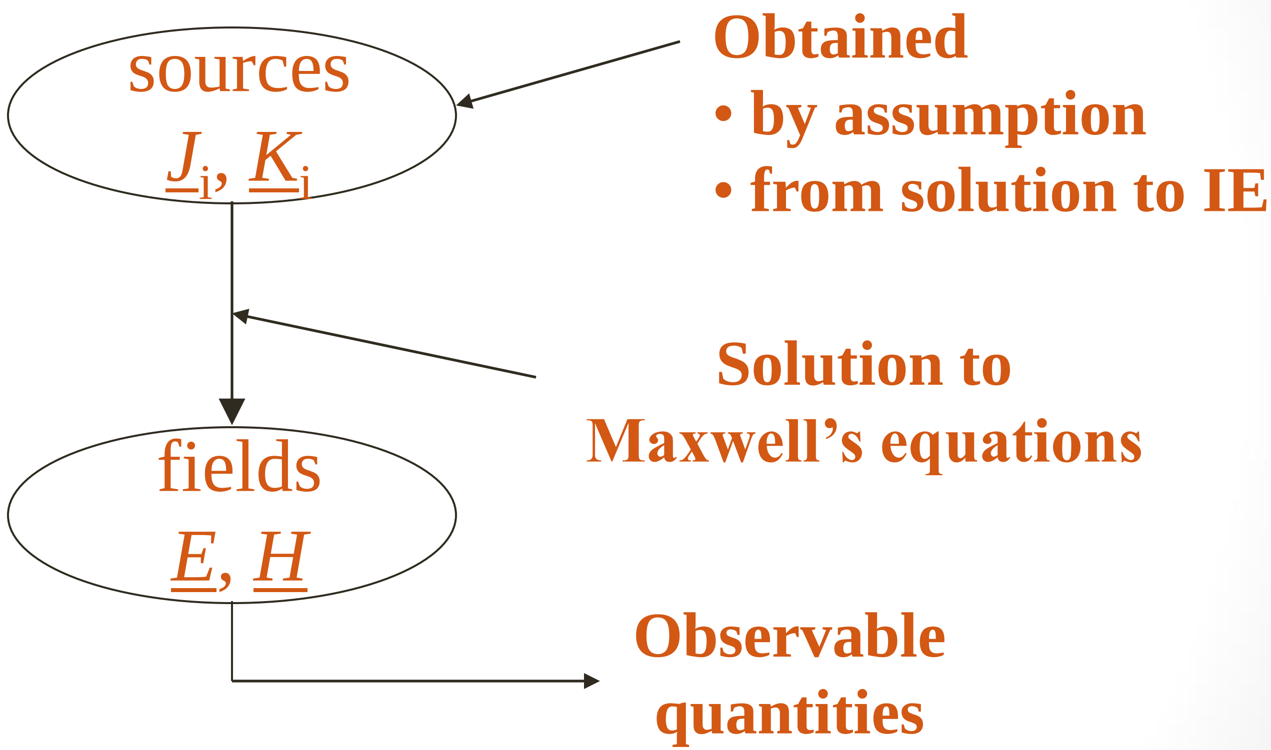
$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

In free space:

$$\underline{B} = \mu_0 \underline{H}$$

$$\underline{D} = \varepsilon_0 \underline{E}$$

Introduction to Electromagnetic Fields



Maxwell's Equations

- ***Maxwell's equations in integral form*** are the fundamental postulates of classical electromagnetics - all classical electromagnetic phenomena are explained by these equations.
- Electromagnetic phenomena include electrostatics, magnetostatics, electromagnetostatics and electromagnetic wave propagation.
- The differential equations and boundary conditions that we use to formulate and solve EM problems are all derived from *Maxwell's equations in integral form*.

Maxwell's Equations

- Various ***equivalence principles*** consistent with Maxwell's equations allow us to replace more complicated electric current and charge distributions with ***equivalent magnetic sources***.
- These *equivalent magnetic sources* can be treated by a generalization of Maxwell's equations.

Maxwell's Equations in Integral Form (Generalized to Include Equivalent Magnetic Sources)

$$\oint_C \underline{E} \cdot d\underline{l} = -\frac{d}{dt}\int_S \underline{B} \cdot d\underline{S} - \int_S \underline{K}_c \cdot d\underline{S} - \int_S \underline{K}_i \cdot d\underline{S}$$

$$\oint_C \underline{H} \cdot d\underline{l} = \frac{d}{dt}\int_S \underline{D} \cdot d\underline{S} + \int_S \underline{J}_c \cdot d\underline{S} + \int_S \underline{J}_i \cdot d\underline{S}$$

$$\oint_S \underline{D} \cdot d\underline{S} = \int_V q_{ev}\, dv$$

$$\oint_S \underline{B} \cdot d\underline{S} = \int_V q_{mv}\, dv$$

Adding the fictitious magnetic source terms is equivalent to living in a universe where magnetic monopoles (charges) exist.

Continuity Equation in Integral Form (Generalized to Include Equivalent Magnetic Sources)

$$\oint_S \underline{J} \cdot d\underline{s} = -\frac{\partial}{\partial t} \int_V q_{ev}\, dv$$

$$\oint_S \underline{K} \cdot d\underline{s} = -\frac{\partial}{\partial t} \int_V q_{mv}\, dv$$

- The *continuity equations* are implicit in Maxwell's equations.

Electric Current and Charge Densities

- $\boldsymbol{J}_c$ = (electric) conduction current density (A/m^2)
- $\boldsymbol{J}_i$ = (electric) impressed current density (A/m^2)
- q_{ev} = (electric) charge density (C/m^3)

Magnetic Current and Charge Densities

- $\boldsymbol{K}_c$ = magnetic conduction current density (V/m^2)
- $\boldsymbol{K}_i$ = magnetic impressed current density (V/m^2)
- q_{mv} = magnetic charge density (Wb/m^3)

Maxwell's Equations in Differential Form (Generalized to Include Equivalent Magnetic Sources)

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} - \underline{K}_c - \underline{K}_i$$

$$\nabla \times \underline{H} = \frac{\partial \underline{D}}{\partial t} + \underline{J}_c + \underline{J}_i$$

$$\nabla \cdot \underline{D} = q_{ev}$$

$$\nabla \cdot \underline{B} = q_{mv}$$

Continuity Equation in Differential Form (Generalized to Include Equivalent Magnetic Sources)

$$\nabla \cdot \underline{J} = -\frac{\partial q_{ev}}{\partial t}$$

$$\nabla \cdot \underline{K} = -\frac{\partial q_{mv}}{\partial t}$$

- The *continuity equations* are implicit in Maxwell's equations.

Electromagnetic Fields in Materials

- In free space, we have:

$$\underline{D} = \varepsilon_0 \underline{E}$$

$$\underline{B} = \mu_0 \underline{H}$$

$$\underline{J}_c = 0$$

$$\underline{K}_c = 0$$

Electromagnetic Fields in Materials

- In a ***simple medium***, we have:

$$\underline{D} = \varepsilon \underline{E}$$

$$\underline{B} = \mu \underline{H}$$

$$\underline{J}_c = \sigma \underline{E}$$

$$\underline{K}_c = \sigma_m \underline{H}$$

- ***linear*** (independent of field strength)
- ***isotropic*** (independent of position within the medium)
- ***homogeneous*** (independent of direction)
- ***time-invariant*** (independent of time)
- ***non-dispersive*** (independent of frequency)

Electromagnetic Fields in Materials

- ε = permittivity = $\varepsilon_r\varepsilon_0$ (F/m)
- μ = permeability = $\mu_r\mu_0$ (H/m)
- σ = electric conductivity = $\varepsilon_r\varepsilon_0$ (S/m)
- σ_m = magnetic conductivity = $\varepsilon_r\varepsilon_0$ (Ω/m)

Maxwell's Equations in Differential Form for Time-Harmonic Fields in Simple Medium

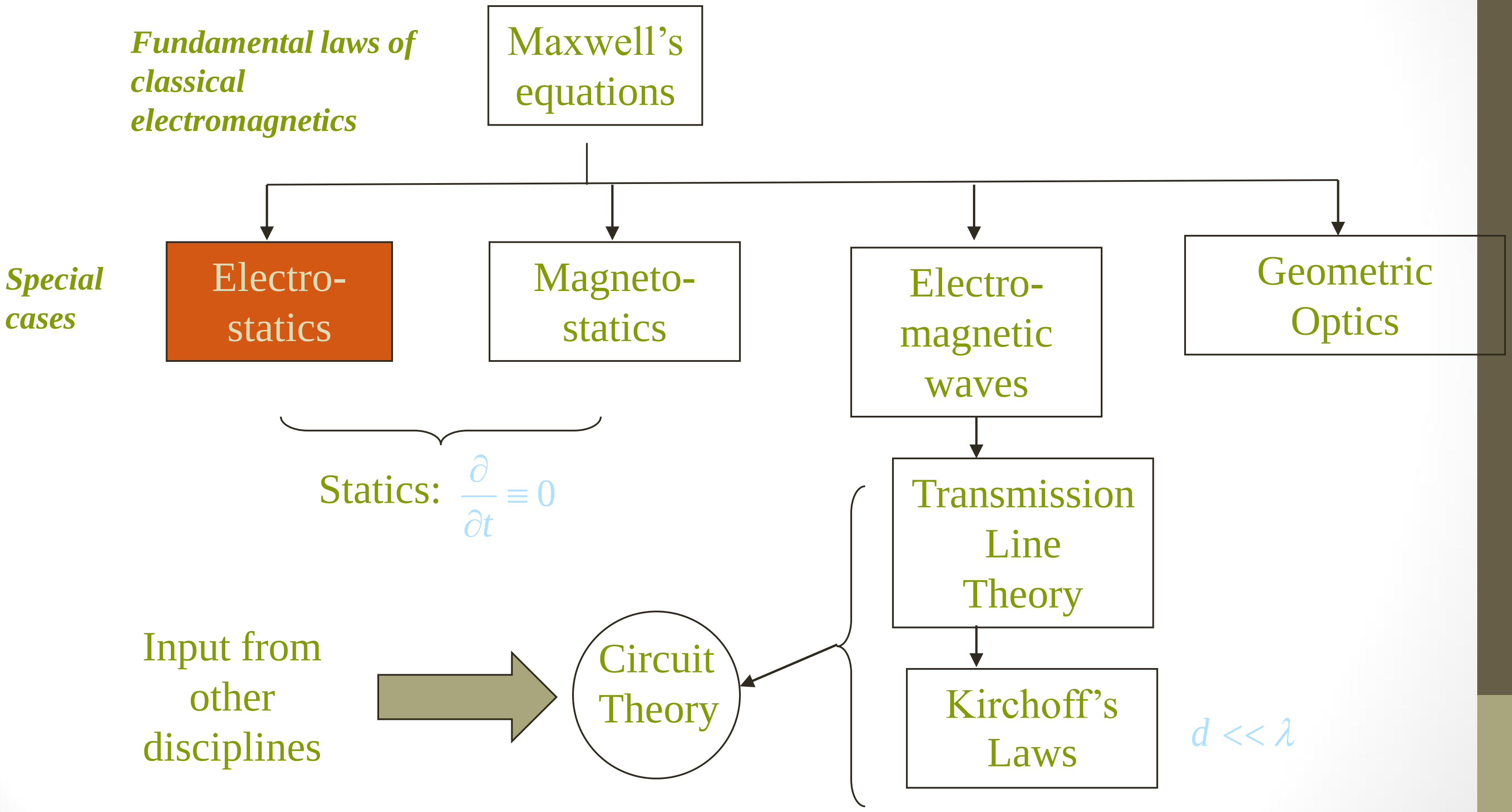
$$\nabla \times \underline{E} = -\left(j\omega\mu + \sigma_m\right)\underline{H} - \underline{K}_i$$

$$\nabla \times \underline{H} = \left(j\omega\varepsilon + \sigma\right)\underline{E} + \underline{J}_i$$

$$\nabla \cdot \underline{E} = \frac{q_{ev}}{\varepsilon}$$

$$\nabla \cdot \underline{H} = \frac{q_{mv}}{\mu}$$

Electrostatics as a Special Case of Electromagnetics



Electrostatics

- ***Electrostatics*** is the branch of electromagnetics dealing with the effects of electric charges at rest.
- The fundamental law of *electrostatics* is ***Coulomb's law***.

Electric Charge

- Electrical phenomena caused by friction are part of our everyday lives, and can be understood in terms of ***electrical charge***.
- The effects of ***electrical charge*** can be observed in the attraction/repulsion of various objects when "charged."
- Charge comes in two varieties called "positive" and "negative."

Electric Charge

- Objects carrying a net positive charge attract those carrying a net negative charge and repel those carrying a net positive charge.
- Objects carrying a net negative charge attract those carrying a net positive charge and repel those carrying a net negative charge.
- On an atomic scale, electrons are negatively charged and nuclei are positively charged.

Modifications to Ampère's Law

- Ampère's Law is used to analyze magnetic fields created by currents:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_o I$$

- But, this form is valid only if any electric fields present are constant in time.
- Maxwell modified the equation to include time-varying electric fields.
- Maxwell's modification was to add a term.

Modifications to Ampère's Law, cont

- The additional term included a factor called the **displacement current**, I_d.

$$I_d = \varepsilon_o \frac{d\Phi_E}{dt}$$

- This term was then added to Ampère's Law.
- This showed that magnetic fields are produced both by conduction currents and by time-varying electric fields.

The general form of Ampère's Law is

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_o (I + I_d) = \mu_o I + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

- Sometimes called Ampère-Maxwell Law